PERSONAL DATA

First Name:	DEBORA
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Born:	Iglesias (CI), 04/04/1985
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CURRENT POSITIONS

Since March 2017	Ricercatore a tempo determinato di tipo A, Dipartimento di Scienze Matematiche, Politecnico di Torino

PREVIOUS POSITIONS

May 2012–April 2016	Assegno di Ricerca, Dipartimento di Matematica e Applicazioni, Università degli Studi di Milano-Bicocca

ACADEMIC HABILITATIONS

February 2016	Qualification Française aux fonctions de *Maître de conférences.* 25ème section.

EDUCATION

March 2012	Ph.D. in Mathematics, Università degli Studi di Milano Thesis–Title: *On the Geometry of Newton operators* Supervisor: Marco Rigoli Defended on March 7th 2012
September 2008	Laurea specialistica (master degree) cum laude in Mathematics Università degli Studi di Cagliari (Cagliari, Italy) Thesis–title: *Alcune proprietà delle applicazioni e delle immersioni biarmoniche* Supervisor: Stefano Montaldo
October 2006	Laurea triennale (bachelor) cum laude in Mathematics

	Università degli Studi di Cagliari (Cagliari, Italy) Thesis–title: *Approssimazione dei numeri irrazionali* Supervisor: Luigi Cerlienco
July 2003	Diploma di Maturità (A–level) with full marks Liceo Scientifico Edoardo Amaldi (Carbonia (CI), Italy)

PUBLICATIONS

[ILPS] D. Impera, J.H Lira, S. Pigola, A.G. Setti
Height estimates for Killing graphs.
Accepted for publication on J. Geom. Anal.
Preliminary version on **arXiv:1612.01257**.

[IP] D. Impera, S. Pigola
On the growth of supersolutions of nonlinear PDE's on exterior domains.
Nonlinear Analysis
Volume **146** (2016), 20–31.
http://doi.org/10.1016/j.na.2016.08.010

[IR2] D. Impera, M. Rimoldi
Rigidity results and topology at infinity of translating solitons of the mean curvature flow.
Commun. Contemp. Math.
Volume **19** (2017), no. 6, 21 pp.
https://doi.org/10.1142/S021919971750002X

[IPS] D. Impera, S. Pigola and A. G. Setti
Potential theory for manifolds with boundary and applications to controlled mean curvature graphs.
Accepted for publication on J. Reine Angew. Math.
doi:10.1515/ crelle-2014-0137
Preliminary version on **arXiv:1303.2853**.

[IR] D. Impera, M. Rimoldi
Stability properties and topology at infinity of f-minimal hypersurfaces.
Geom. Dedicata
Volume **178** (2015), 21–47.
doi:10.1007/s10711-014-9999-6

[GI] S. C. García-Martínez and D. Impera
Height estimates and half-space theorems for spacelike hypersurfaces in generalized Robertson-Walker spacetimes.
Differential Geom. Appl.
Volume **32** (2014), 46–67.
doi:10.1016/j.difgeo.2013.10.017

[GIR] S. C. García-Martínez, D. Impera, M. Rigoli
A sharp height estimate for compact hypersurfaces with constant k-mean curvature in warped product spaces.
Proc. Edinb. Math. Soc., available on CJO2014.
doi:10.1017/S0013091514000157

[AIR] L. J. Alías, D. Impera, M. Rigoli
Hypersurfaces of constant higher order mean curvature in warped products.

TRANS. AMER. MATH. SOC.
Volume **365** (2013), no. 2, 591–621.

[I] D. IMPERA
Comparison Theorems in Lorentzian Geometry and applications to spacelike hypersurfaces
J. GEOM. PHYS.
Volume **62** (2012) no. 2, 412–426.
doi:10.1016/j.geomphys.2011.11.004

[AIR2] L. J. ALÍAS, D. IMPERA, M. RIGOLI
Spacelike hypersurfaces of constant higher order mean curvature in generalized Robertson-Walker spacetimes
MATH. PROC. CAMBRIDGE PHILOS. SOC.
Volume **152** (2012) 365–383.
doi:10.1017/S0305004111000697
(see also the erratum on Math. Proc. Cambridge Philos. Soc.)

[IMR] D. IMPERA, L. MARI, M. RIGOLI
Some geometric properties of hypersurfaces with constant r-mean curvature in Euclidean space
PROC. AMER. MATH. SOC.
Volume **139** (2011), no. 6, 2207–2215.
(see also the corrigenda on Proc. Amer. Math. Soc.)

[IM] D. IMPERA, S. MONTALDO
Totally biharmonic submanifolds
Differential geometry, 232–246, World Sci. Publ., Hackensack, NJ, 2009.

PREPRINTS

[I2] D. IMPERA
Rigidity and gap results for low index properly immersed self–shrinkers in $\mathbb{R}^{m+1}$.
Submitted for publication. Preliminary version on **arXiv:1408.3479**.

TALKS, SEMINARS AND POSTERS

July 2017 UNIVERSITÄT KONSTANZ, KONSTANZ, GERMANY
Oberseminars Partielle Differentialgleichungen
Invited seminar talk:
"Rigidity results and topology at infinity of translating solitons of the mean curvature flow."

January 2016 INSTITUT HENRI POINCARÉ, PARIS, FRANCE
Seminaire de Géométrie
Invited seminar talk:
"Topology at infinity of translating solitons of the mean curvature flow."

September 2015 UNIVERSITÀ DI SIENA, SIENA, ITALY
XX Congresso dell'Unione Matematica Italiana
Contributed talk:
"Solitoni traslati per il flusso di curvatura media in $\mathbb{R}^{n+1}$*"*

June 2015	FREIE UNIVERSITÄT, BERLIN, GERMANY *Topics on Geometric Analysis* Invited seminar talk: *"Rigidity results and topology at infinity of translating solitons of the mean curvature flow."*
February 2015	UNIVERSITÀ DEGLI STUDI DI CAGLIARI, ITALY *Seminari di Matematica 2015* Invited seminar talk: *"Alcuni aspetti della teoria del potenziale su varietà con bordo e applicazioni a grafici con curvatura media controllata"*
October 2014	LEVICO TERME, TRENTO, ITALY *Progressi Recenti in Geometria Reale e Complessa - IX* Invited talk: *"Some aspects of potential theory on complete manifolds with boundary and applications to controlled mean curvature graphs"*
July 2014	UNIVERSIDAD DEL PAÍS VASCO, BILBAO, SPAIN *First Joint International Meeting of the Italian and Spanish Mathematical Societies RSME-SCM-SEMA-SIMAI-UMI* Invited talk: *"Potential Theory for manifolds with boundary and applications to controlled mean curvature graphs"*
February 2014	UNIVERSIDADE FEDERAL DO CEARÀ, FORTALEZA, BRASIL *VII Workshop on Geometric Analysis* Invited talk: *"Stability properties and topology at infinity of f-minimal hypersurfaces"*
May 2013	UNIVERSITÀ DI MILANO–BICOCCA, ITALY *Seminari di Analisi Armonica* Seminar: *"Stime di altezza per grafici a curvatura media costante"*
September 2011	UNIVERSIDAD DE GRANADA, SPAIN *VI International Meeting on Lorentzian Geometry* Contributed talk: *"Spacelike hypersurfaces of constant higher order mean curvature in generalized Robertson-Walker spacetimes"*
February 2011	UNIVERSIDAD DE GRANADA, SPAIN *Spanish–Japanese Workshop on Differential Geometry in Granada* Poster: *"Hypersurfaces of constant higher order mean curvature in warped products"*
December 2010	UNIVERSITÀ DEGLI STUDI DI CAGLIARI, ITALY *Seminari di Geometria* Seminar: *"Superfici con curvatura media di ordine k costante in spazi dotati di metriche distorte"*

INVITATIONS

Jan.-April 2016	Université Paris-Est Marne la Vallée, France (3 months) Winner of a INDAM scholarship for a research period abroad
February 2015	Università di Cagliari, Cagliari, Italy (1 week)
February 2014	Universidade Federal do Cearà, Fortaleza, Brasil (2 weeks)
Sept.-Dec. 2012	Institut Henri Poincaré, Paris, France (3 months) Guest for the research trimester in *Conformal and Kälher Geometry*

REFEREE ACTIVITY

Referee for: Annali dell'Università di Ferrara
Geometriae Dedicata
Journal of Mathematical Analysis and Applications
Kodai Mathematical Journal
The Scientific World Journal

OTHER ACTIVITIES

Coordinator of the INdAM-GNAMPA Project 2014:
Proprietà analitiche e spettrali
di varietà pesate e applicazioni

Member of the organizing committee of the workshop:
A Geometry Day in Como 2015
Como, Italy, Jan 10, 2014
Talks by: Z. Djadli, A. Malchiodi, L. Mari,
B. Nelli, R. Paoletti.

Member of the organizing committee of the workshop:
A Geometry Day in Como 2014
Como, Italy, Jan 10, 2014
Talks by: F. Bonsante, F. Coda-Marques, I. Holopainen, N. Gigli,
C. Mantegazza, M. Rigoli.

Member of the organizing committee of the school:
Geometric Analysis on Riemannian and singular metric spaces
Como, Italy, Sept 30-Oct 5, 2013
Mini-courses by: S. Alexander, G. Carron, E. Hebey, U. Lang, A. Neves
Scientific Committee: G. Besson, S. Pigola, A. Setti, M. Troyanov.

Organizer of the workshop:
A Geometry Day in Bicocca
Como, Italy, Sept 27, 2013

CONFERENCES, WORKSHOPS AND SUMMER SCHOOLS

September 2015	Università di Siena, Siena, Italy *XX Congresso dell'Unione Matematica Italiana*

March 2015	GOETHE-UNIVERSITY, FRANKFURT, GERMANY *Flowers and Friends in Frankfurt- A workshop in Geometric Analysis*
October 2014	LEVICO TERME (TN), ITALY *Progressi Recenti in Geometria Reale e Complessa - IX*
September 2014	VILLASIMIUS (CA), ITALY *New trends in Differential Geometry 2014*
July 2014	UNIVERSIDAD DEL PAÍS VASCO, BILBAO, SPAIN *First Joint International Meeting of the Italian and* *Spanish Mathematical Societies RSME-SCM-SEMA-SIMAI-UMI*
February 2014	UNIVERSIDADE FEDERAL DO CEARÀ, FORTALEZA, BRASIL *VII Workshop on Geometric Analysis*
January 2014	UNIVERSITÀ DEGLI STUDI DELL'INSUBRIA, ITALY *A geometry day in Como*
October 2013	LAKE COMO SCHOOL FOR ADVANCED STUDIES, COMO, ITALY *School in Geometric Analyisis* *Geometric Analysis on Riemannian and singular metric spaces*
June 2013	UNIVERSIDAD DE GRANADA, SPAIN *Variational problems and Geometric PDE's*
Feb.-Mar. 2013	SCUOLA NORMALE SUPERIORE, PISA, ITALY *Workshop su varietà reali e complesse:* *geometria, topologia e analisi armonica*
January 2013	UNIVERSITÀ DEGLI STUDI DELL'INSUBRIA, ITALY *A geometry day in Como*
Sept.-Dec. 2012	INSTITUT HENRI POINCARÉ, PARIS, FRANCE *Research trimester in Conformal and Kälher geometry*
June 2012	ICTP, TRIESTE *ICTP-ESF School and Conference on Geometric Analysis*
May 2012	UNIVERSITÀ DI MILANO BICOCCA, ITALY *Geometria in Bicocca*
September 2011	UNIVERSIDAD DE GRANADA, SPAIN *VI International Meeting on Lorentzian Geometry*
May 2011	UNIVERSITÀ DI MILANO BICOCCA, ITALY *Geometria in Bicocca*
April 2011	CENTRO DE GIORGI, PISA, ITALY *Ricci Solitons Days in Pisa 2011*
February 2011	UNIVERSITÀ DI MILANO BICOCCA, ITALY 1st Bicocca HART *Harmonic Analysis and Related Topics*

February 2011	UNIVERSIDAD DE GRANADA, SPAIN *Spanish–Japanese Workshop on Differential Geometry*	
November 2010	UNIVERSIDAD DE CORDOBA, SPAIN *International Meeting on Differential Geometry*	
September 2010	VERBANIA, ITALY Riemann International School of Mathematics *Nonlinear Differential Equations*	
June–July 2010	UNIVERSIDAD DE GRANADA, SPAIN *Santaló summer school in Geometric Analysis*	
May 2010	DRESDEN UNIVERSITY OF TECHNOLOGY, GERMANY The 8th AIMS Conference on *Dynamical Systems, Differential Equations and Applications*	
September 2009	UNIVERSITÀ DI CAGLIARI, ITALY *A harmonic map fest*	
June 2009	CENTRO DE GIORGI, PISA, ITALY Research Trimester on *Geometric flows and Geometric Operators*	
April 2009	VERBANIA, ITALY Riemann International School of Mathematics *Advances in number theory and geometry*	
August 2007	PERUGIA, ITALY *SMI (Scuola Matematica Interuniversitaria)* Differential Geometry–Algebraic Geometry	

PROFESSIONAL EXPERIENCE

March–June 2017	Politecnico di Torino Pianificazione (Triennale) Teaching assistant (20 hours) for the following course: “Calcolo”	TORINO, ITALY
March–June 2015	Università degli Studi di Milano-Bicocca Matematica (Triennale) Teaching assistant (24 hours) for the following course: “Geometria 2”	MILANO, ITALY
March–June 2014	Università degli Studi di Milano-Bicocca Matematica (Triennale) Teaching assistant (24 hours) for the following course: “Geometria 2”	MILANO, ITALY
March–June 2012	Politecnico di Milano Ingegneria Industriale Teaching assistant (48 hours) for the following course: “Analisi 2 e Geometria”	MILANO, ITALY
March–June 2012	Politecnico di Milano	MILANO, ITALY

	Ingegneria Gestionale Teaching assistant (20 hours) for the following course: “Probabilità e Statistica Matematica”	
March–June 2012	Università degli Studi dell’Insubria, Matematica (Triennale) Teaching assistant (10 hours) for the following course: “Analisi 2”	Como, Italy
Oct. 2011–Jan. 2012	Università degli Studi dell’Insubria, Scienze dell’Ambiente e della Natura Teaching assistant (8 hours) for the following course: “Matematica”	Como, Italy
Oct. 2011–Jan. 2012	Politecnico di Milano Ingegneria Meccanica e Aerospaziale Teaching assistant (48 hours) for the following course: “Analisi 1 e Geometria”	Milano, Italy
October 2009	Università degli Studi di Milano, Facoltà di Scienze MM.FF.NN. Teaching assistant (21 hours) for the MINIMAT course (remedial classes of Mathematics) addressed to freshman students of the Facoltà di Scienze MM. FF. NN. and of the 5 courses of the Biotechnology degree.	Milano, Italy
Oct. 2008–Jan. 2009	Università degli Studi di Cagliari Dipartimento di Matematica Teaching assistant (32 hours) for the following course: “Geometria 1”	Cagliari, Italy
April 2008	Istituto Comprensivo di Santadi Involvement in the organization of the exhibition ‘Do you play mathematics?’ and in the activities as expert guide for school students and other visitors at the Cittadella dei Musei in Cagliari.	Santadi (CI), Italy
April 2008	Istituto Comprensivo di Santadi Supervision, training of the guides and involvement in the organization of the exhibition ‘Ma-te-logic@-mente’ and in the activities as expert guide for school students and other visitors at the Centro Sociale in Santadi (CI).	Santadi (CI), Italy
March-April 2008	Istituto Comprensivo di Santadi Involvement in the project ‘Ma-te-logic@-mente’ as teacher encharged by the Università di Cagliari.	Santadi (CI), Italy
April 2007	Università degli Studi di Cagliari Facoltà di Scienze MM.FF.NN. Involvement as expert guide in the organization of the exhibition of Mathematics during the ‘settimana di cultura scientifica’ organized at the Dipartimento di Matematica	Cagliari, Italy

of the Facoltà di Scienze MM.FF.NN.

RESEARCH ACTIVITY

So far, my research activity has mainly dealt with geometric PDE's and their application to the study of hypersurfaces. In particular, I've been focusing on the following topics:

- **Hypersurfaces of constant mean curvature in Riemannian manifolds** ([IPS]).
In [IPS] we focus on the geometry of graphs with prescribed mean curvature inside a Riemannian product of the type $\mathbb{P} \times \mathbb{R}$. The main tools we use toward this aim are versions for non-compact manifolds with boundary of the classical Ahlfors maximum principle and of the so called Kelvin Nevanlinna-Royden criterion of parabolicity. Recall that a manifold M with empty boundary is parabolic if every solution of the problem

$$\begin{cases} \Delta u \geq 0 & \text{on } M \\ \sup_M u < +\infty \end{cases}$$

is the constant function $u \equiv \sup_M u$. For what concern manifolds with non-empty boundary, a quick check at the literature shows that there are many definitions of parabolicity and they are in a certain hierarchy, so one has to make a choice. In view of our geometric purposes we decided to follow the more traditional path, that, from the stochastic viewpoint, translates into the property that the reflected Brownian motion be recurrent. The definition we decided to adopt is then the following

> An oriented Riemannian manifold M with boundary $\partial M \neq \emptyset$ is said to be parabolic if for every $u \in C^0(M) \cap W^{1,2}_{loc}(M)$,
>
> $$\begin{cases} \Delta u \geq 0 & \text{on } M \\ \dfrac{\partial u}{\partial \nu} \leq 0 & \text{on } \partial M \\ \sup_M u < +\infty \end{cases} \quad \Rightarrow \quad u \equiv \text{const.}$$

As observed by Grigor'yan in [Gr2], the recurrence of the Brownian motion for manifolds without boundary can be characterized in terms of fundamental solutions to the Laplace equation, superharmonic functions, capacities, the heat kernel, the Liouville property for certain Schrödinger equations, volume growth, function theoretic tests (Khas'minskii criterion), Kelvin-Nevanlinna-Royden criterion and many other geometric and potential-analytic properties. In [Gr1] he investigated if similar characterizations hold also for the reflected recurrent Brownian motion and was able to extend some of the geometric and potential-analytic properties listed above to manifolds with boundary, suggesting that his notion of parabolicity may be the right one to consider in this context. In [IPS] we move some step in this direction and we extend to non-compact Riemannian manifolds with boundary the use of two important tools in the geometric analysis of compact spaces, namely, the integration by parts and the weak maximum principle for subharmonic functions. Using this tools we are able to obtain height estimates for constant mean curvature graphs parametrized over unbounded domains in a complete manifold and slice type results for graphs whose superlevel sets have finite volume. To be more precise, recall that given a (not necessarily open) domain $\Omega \subset \mathbb{P}^n$ and a smooth function $u : \Omega \to \mathbb{R}$, the graph of u over the a domain Ω is the following subset of $\mathbb{P} \times \mathbb{R}$:

$$\Sigma_u = \{(x, u(x)) \in \mathbb{P} \times \mathbb{R} | x \in \Omega\}.$$

A graph Σ_u has constant mean curvature H if and only if u is a solution of the quasi-linear PDE

$$-nH = \operatorname{div}\left(\frac{\nabla u}{\sqrt{1 + |\nabla u|^2}}\right), \tag{1}$$

where div is the divergence in $\mathbb{P}$.
If Ω is a bounded domain, height estimates for constant positive mean curvature graphs with boundary on a slice can be easily obtained as an application of the classical maximum principle, exploiting the fact that the function u is subharmonic with respect to the induced metric on Σ_u and the angle function $\Theta := \langle N, \partial/\partial t\rangle$ is superharmonic. Here N is the unit normal vector field and $\partial/\partial t$ is the coordinate vector field on $\mathbb{R}$. In the unbounded setting, once guaranteed the parabolicity of the graph with respect to the induced metric, we are able to extend these estimates using the counterpart of the classical maximum principle in the non-compact case, that is the Ahlfors maximum principle.

As for the slice-type theorems, consider a graph $\Gamma_u : \mathbb{P} \to \mathbb{R}\times\mathbb{P}$, $\Gamma_u(\mathbb{P}) = \Sigma_u$, of non-positive mean curvature $H(x) \leq 0$. If $\mathbb{P}$ is compact and either $\Gamma_u(\partial\mathbb{P}) \subset \mathbb{P}_0$ or $\partial u/\partial\nu = 0$ on $\partial\mathbb{P}$, then, as an application of the divergence theorem, it is not difficult to prove that Γ_u is a slice of $\mathbb{R}\times\mathbb{P}$. If $\mathbb{P}$ is non-compact and $\partial\mathbb{P} \neq \emptyset$, a global divergence theorem for L^1 vector fields with L^1 divergence holds provided $\mathbb{P}$ is parabolic (Kelvin-Nevanlinna-Royden criterion). Under a volume assumption on $\mathbb{P}$ implying parabolicity and an upper bound for the function u defining the graph, it is then possible to apply the Kelvin-Nevanlinna-Royden criterion and to obtain the same characterization of horizontal slices.

- f**–minimal hypersurfaces**([IR]).
Many problems in geometric analysis lead to consider Riemannian manifolds endowed with a measure that has a smooth positive density with respect to the Riemannian one. This turns out to be compatible with the metric structure of the manifold and the resulting spaces take the name of weighted manifolds, also known in literature as manifolds with density. Weighted manifolds first arose in the study of diffusion processes on manifolds in works of D. Bakry and M. Émery, [BE], and were intensively studied in recent years; see e.g. the seminal works of F. Morgan, [Mo-notices], and G. Wei, W. Wylie, [WW]. A weighted manifold is a triple $M^n_f = (M^n, \langle\, ,\,\rangle, e^{-f}d\mathrm{vol})$, where $(M^n, \langle\, ,\,\rangle)$ is a complete n–dimensional Riemannian manifold, $f \in C^\infty(M)$ and $d$vol denotes the canonical Riemannian volume form on M. The geometry of weighted manifolds is visible in the weighted metric structure, i.e., in the weighted measure of (intrinsic) metric objects, and it is controlled by suitable concepts of curvature adapted to the density of the measure. In [BE], it was introduced an important generalization of Ricci curvature in this setting, known as Bakry–Emery Ricci tensor and defined as
$$^M\mathrm{Ric}_f = {}^M\mathrm{Ric} + {}^M\mathrm{Hess}(f).$$
Following M. Gromov, [G], if we consider an isometrically immersed orientable hypersurface Σ^n in the weighted manifold M^{n+1}_f, we can define a generalization of the mean curvature vector field as $\mathbf{H}_f = \mathbf{H} + \left(\overline{\nabla} f\right)^\perp$. Since the weighted structure on M induces a weighted structure on Σ we can consider the variational problem for the weighted area functional. From variational formulae, one can see that Σ is f–minimal, namely a critical point of the weighted area functional, if and only if $\mathbf{H}_f$ vanishes identically. Clearly minimal hypersurfaces are a particular case of f–minimal hypersurfaces corresponding to the case $f \equiv const$. Moreover, self–shrinkers of the mean curvature flow are important examples of f–minimal hypersurfaces in the Euclidean space with the Gaussian density $e^{-\frac{|x|^2}{2}}$. The research on f–minimal hypersurfaces has just started and it has been already approached by many authors, see e.g. [CMZ], [Esp], [F]. Much efforts have been devoted to the study of the stability properties. These are taken into account by spectral properties of the weighted Jacobi operator $L_f = -\Delta_f - \left(|\mathbf{A}|^2 + \overline{\mathrm{Ric}}_f(\nu,\nu)\right)$. The most up to date result, proved by X. Cheng, T. Mejia, and D. Zhou, [CMZ], states that there exist no L_f–stable complete f–minimal hypersurfaces Σ immersed in a complete weighted Riemannian manifold M_f with $\overline{\mathrm{Ric}}_f \geq k > 0$, provided $\mathrm{vol}_f(\Sigma) < +\infty$. In [IR] we generalized this result considering growth conditions on the intrinsic weighted volume of geodesic balls. Furthermore, in the instability case, exploiting the oscillatory behavior of solutions of some ODEs that naturally arise in this setting, we investigate general geometric restrictions for the finiteness of the weighted index of the f–minimal hypersurface, that is, the maximum dimension of the linear space of compactly

supported deformations that decrease the weighted area up to second order. Moreover, exploiting a weighted version of a finiteness result and the adaptation to this setting of Li–Tam theory, we investigated the topology at infinity of f–minimal hypersurfaces. On the way, we prove a new comparison result in weighted geometry and we provided a general weighted L^1–Sobolev inequality for hypersurfaces in Cartan–Hadamard weighted manifolds, satisfying suitable restrictions on the weight function and on its radial derivative.

- **Self–shrinkers of the mean curvature flow**([I2])
By a self–shrinker of the mean curvature flow we mean a connected, isometrically immersed hypersurface $x : \Sigma^m \to \mathbb{R}^{m+1}$ whose mean curvature vector field $\mathbf{H}$ satisfies the equation $x^\perp = -\mathbf{H}$. Self–shrinkers play an important role in the study of mean curvature flow, since they describe all possible blow ups at a given singularity. Standard examples of self–shrinkers are the hyperplanes through the origin of $\mathbb{R}^{m+1}$, the sphere $\mathbb{S}^m(\sqrt{m})$ and the cylinders $\Sigma = \mathbb{R}^{m-k} \times \mathbb{S}^k(\sqrt{k})$ for some $1 \le k \le m-1$.

  It is well–known that self–shrinkers in $\mathbb{R}^{m+1}$ can be viewed as f–minimal hypersurfaces, that is, critical points of the weighted area functional
$$\mathrm{vol}_f(\Sigma) = \int_\Sigma e^{-f} d\mathrm{vol}_\Sigma,$$
where $f = |x|^2/2$. Moreover, we say that a self–shrinker is f–stable if it is a local minimum of the weighted area functional for every compactly supported normal variation. In the instability case, it makes sense to investigate the Morse index, that is, roughly speaking, the maximum dimension of the linear space of compactly supported deformations that decrease the weighted area up to second order. From the spectral viewpoint there are many analogies between properly immersed self–shrinkers in the Euclidean space and complete minimal surfaces in the sphere. For instance, it was proved by Colding and Minicozzi, [CoMi], that every complete properly immersed self–shrinker every complete properly immersed self–shrinker is necessarily f–unstable. In the instability case, rigidity results have been proved by Hussey, [Hu], under the additional assumption of embeddedness. More precisely, he showed that if a complete properly embedded self–shrinker in $\mathbb{R}^{m+1}$ has Morse index 1, then it has to be a hyperplane through the origin. Furthermore, he also proved that if the self-shrinker is not a hyperplane through the origin, then the Morse index jumps and it has to be at least $m+2$, with equality if and only if the self–shrinker is a cylinder $\mathbb{R}^{m-k} \times \mathbb{S}^k(\sqrt{k})$ for some $1 \le k \le m$. In [I2], exploiting the link between stability properties of self–shrinkers and spectral properties of a suitable weighted Schrödinger operator, as well as some basic identities which naturally involve the weighted Laplacian of the self–shrinker, we show that, in fact, hyperplanes through the origin and cylinders remains the only hypersurfaces, among the wider family of properly immersed self–shrinkers in $\mathbb{R}^{m+1}$, having Morse index 1 and $m+2$ respectively. Moreover, we also show that, except for them, every properly immersed self–shrinker has Morse index strictly bigger than $m+2$.

- **Translators of the mean curvature flow**([IR2]).
By a translator of the mean curvature flow we mean a connected isometrically immersed complete hypersurface $x : \Sigma^m \to \mathbb{R}^{m+1}$ whose mean curvature vector field $\mathbf{H}$ satisfies the equation $\mathbf{H} = v^\perp$ for some fixed unit length vector $v \in \mathbb{R}^{m+1}$. These hypersurfaces correspond to translating solitons of the mean curvature flow, and play a key role in the study of slowly forming singularities. Translators of the MCF turn out to be f–minimal hypersurfaces in the Euclidean space $\mathbb{R}^{m+1}$ with the density $e^{\langle v,x\rangle}$. In collaboration with Michele Rimoldi, in [IR2], we highlighted how the realm of weighted manifolds and f–minimal hypersurfaces can naturally give strong enough characterization and topological results for translators. First, we proved a rigidity theorem for f–stable translators under a weighted L^2–condition on the norm of the second fundamental form. This result, in particular, permits to strengthen previous rigidity results, see [Sc]. Concerning the latters, exploiting in an essential way the correspondence, discovered by K. Smoczyk, [Sm], between translators of the mean curvature flow in $\mathbb{R}^{m+1}$ and minimal hypersurfaces in the manifold $\mathbb{R}^{m+1} \times \mathbb{R}$, endowed with a suitable

warped product metric, we obtain the validity of a weighted $(m+1)$–dimensional L^1 Sobolev inequality on translators. The validity of this inequality permits to obtain very neat results on the topology at infinity of f–stable translators and translators with finite f–index. The main tools here are again weighted versions of the Li-Tam theory and of an abstract finiteness results, which were developed in [IR1]. Moreover, under the geometric assumption that the translator is contained in a halfspace determined by an hyperplane orthogonal to the translating direction v, we were able to guarantee the validity of the standard (m–dimensional) L^1–Sobolev inequality. This permit to get topological conclusion on translators also in case, instead of assuming the finiteness of the f–index, we are asking that the hypersurface has finite weighted total curvature.

- **Hypersurfaces of constant higher order mean curvature in Riemannian manifolds** ([GIR], [AIR], [IMR]).
The natural generalization of mean (and scalar) curvature for an n-dimensional hypersurface are the k-mean curvatures H_k, $k = 1, \dots, n$, that are defined via the elementary symmetric functions of the principal curvatures of the immersion. More precisely, let Σ^n be a connected oriented Riemannian n-manifold and let $f : \Sigma^n \to M^{n+1}$ be an isometric immersion of Σ^n into an orientable Riemannian $(n + 1)$-manifold M^{n+1}. We will denote by A the linear operator associated to the second fundamental form of the immersion and by N the unit normal vector field globally defined on Σ^n. Let $k_1, ..., k_n$ be the *principal curvatures* of the immersion and denote by S_k the k-th symmetric function of the principal curvatures, defined as
$$S_0 = 1, \quad S_k = \sum_{i_1<\ldots<i_k} k_{i_1} \cdots k_{i_k},\ 1 \le k \le n, \quad S_k = 0,\ k > n.$$
The *k-mean curvature* of f is then defined as
$$S_k = \binom{n}{k} H_k.$$
Associated with the higher order mean curvatures there is a family of operators, the so-called *Newton transformations*, which are related to the second fundamental form A and are inductively defined as
$$P_0 = I, \quad P_k = S_k I - AP_{k-1},\ 1 \le k \le n.$$
Associated to each Newton transformation P_k of an immersion $f : \Sigma^n \to M^{n+1}$, there is a second order differential operator $L_k : C^\infty(\Sigma) \to C^\infty(\Sigma)$ defined by
$$L_k u = \mathrm{Tr}(P_k \circ \mathrm{hess} u).$$
As already said, immersions of constant mean curvature are critical points for the variational problem of minimizing the area functional for compactly supported volume-preserving variations. When the ambient space has constant sectional curvature c, also the higher order mean curvatures come out from a variational problem. Indeed it is not difficult to prove that constant k-mean curvature hypersurfaces are critical points of a generalized area functional for compactly supported volume-preserving variations (see [Re], [Ro] or [BC] for more details). Furthermore, if one is interested in studying when this critical points are also minima of the functional, calculating the second variational formula one sees that the expression for the Jacobi operators involves the operators L_k previously introduced.
(i) In [IMR] we exploit recent results in [BMR] to analyze the stability of the differential operator L_{k-1} associated with the $(k-1)$-th Newton tensor of f. Studying a certain Cauchy problem and its oscillatory solutions we estimate the first eigenvalue $\lambda^{T_{k-1}}$ of the Jacobi operator T_{k-1} (corresponding to H_k constant) on the exterior of concentric balls with increasing radii, thus extending previous works of M. P. do Carmo and D. T. Zhou [dCZ] and M. F. Elbert [E]. As a first application we prove that the normal map N of a complete (connected and non-compact) oriented hypersurface Σ^n immersed in the Euclidean space $\mathbb{R}^{n+1}$ with $H_k > 0$, $1 \le k \le n$, meets each equator of $\mathbb{S}^n$ infinitely many times, provided

the second fundamental form A is (positive) definite at some $p \in \Sigma$ and the growth of the integral of H_{k-1} over geodesic spheres is somehow controlled by that of H_1. As a second application we prove that when $H_k \equiv 0$, the tangent envelope of Σ coincides with the entire space $\mathbb{R}^{n+1}$, provided $rank(A) > k-1$ at all points and we assume a similar growth control on the integral of H_{k-1}.
(ii) In [AIR] we deal with uniqueness results for hypersurfaces of constant higher order mean curvature immersed in suitable ambient manifolds. To do that, in the spirit of the Alexandrov Theorem [A1] [A2], we consider manifolds with a large class of embedded umbilical hypersurfaces of constant mean curvatures and we then look for geometric condition that force an immersed compact or complete hypersurface of constant k-mean curvature, $2 \le k \le n$, to be one of those already classified, generalizing results of [AD1], [AD2]. As pointed out by Montiel in [Mo1], a natural class of ambient manifolds to consider is that of warped products $M^{n+1} := \mathbb{R} \times_\rho \mathbb{P}^n$, where $\mathbb{P}^n$ is a complete n-dimensional Riemannian manifold. Indeed, each leaf $\mathbb{P}_t = \{t\} \times \mathbb{P}^n$ of the foliation $t \to \mathbb{P}_t$ of M^{n+1} by complete hypersurfaces is totally umbilical and has constant k-mean curvature. The idea is that, since the geometry of these spaces is completely determined by the warping function and by the geometry of the fiber, appropriate conditions on them force the hypersurface to be a slice. We overcame the natural technical problems that appear when passing from the first order to the higher order mean curvature obtaining a generalized version of Omori-Yau maximum principle for trace-type elliptic operators that applies to the operators L_{k-1}. Moreover, this principle is a useful tool that can help to solve other problems related to the topic.
(iii) In [GIR] we deal with height estimates for compact hypersurfaces of constant higher order mean curvature with boundary. In recent years height estimates for constant mean curvature graphs have been studied by several authors, since they are intimately related to important properties of the geometry of submanifolds. The first result in this direction is due by Heinz [He] who proved that a compact graph of positive constant mean curvature H in Euclidean 3-space with boundary on a plane can reach at most height $1/H$ from the plane. More recently an optimal bound was also obtained for compact graphs and also for compact embedded surfaces with constant mean curvature and boundary on a plane in the 3-dimensional hyperbolic space by N. Korevaar, R. Kusner, W. Meeks and B. Solomon [KKMS]. In the case of a Riemannian product $\mathbb{R} \times \mathbb{P}^2$, with $\mathbb{P}^2$ any Riemannian surface, height estimates were exhibited by Hoffman, de Lira and Rosenberg in [HLR] and by Aledo, Espinar and Gálvez in [AEG]. In [Ro], Rosenberg extended the previous results to the case of constant higher order mean curvature. Namely, height estimates are proven for compact hypersurfaces with positive constant k-mean curvature H_k embedded either into the Euclidean or the Hyperbolic space. Later, the same author and Cheng, [CR], found height estimates for compact vertical graphs with positive constant k-mean curvature in the product manifold $\mathbb{R} \times \mathbb{P}$ and boundary on a slice, that is, on a submanifold of the form $\mathbb{P}_\tau = \{\tau\} \times \mathbb{P}$ for some $\tau \in \mathbb{R}$. Finally, Alías and Dajczer in [AD2] gave height estimates in the case of compact hypersurfaces of positive constant mean curvature immersed into general warped product spaces and boundary on a slice, generalizing for $k = 1$, the previous results obtained by Cheng and Rosenberg. In [GIR] we try to complete the picture described above extending the results of Alías and Dajczer to the case of compact hypersurfaces of constant positive k-mean curvature, $2 \le k \le n$, in warped product spaces. Finally observe that, in [CR] height estimates were used to obtain geometric properties of properly embedded hypersurfaces without boundary. More precisely, Cheng and Rosenberg proved that a hypersurface of constant k-mean curvature properly embedded in a product $\mathbb{R} \times \mathbb{P}$, where $\mathbb{P}$ is a compact manifold with non-negative sectional curvature can not lie in a half-space. In this circle of ideas, we apply our height estimates in order to prove non-existence results for properly immersed complete hypersurfaces without boundary in pseudo-hyperbolic spaces and contained in a half-space.

- **Spacelike hypersurfaces of constant higher order mean curvature in Lorentzian manifolds**([I], [AIR2], [GI]).
  The study of spacelike hypersurfaces in Lorentzian ambient spaces is a topic of increasing interest in recent years. Since the proof of the Calabi-Bernstein theorem for maximal hyper-

surfaces given by Cheng and Yau in 1970, many interesting results have been produced by several authors in different contexts, including different Bernstein type results for maximal and constant mean curvature hypersurfaces.

Let $f : \Sigma^n \to M^{n+1}$ be a spacelike hypersurface isometrically immersed into a spacetime M. We recall that a hypersurface is said to be *spacelike* if the induced metric is positive definite (that is, it is a Riemannian manifold with respect to the induced metric). Since M is time-oriented, there exists a unique future-directed timelike unit normal field N globally defined on Σ. We will refer to that normal field as the future-pointing Gauss map of the hypersurface. Similarly to the Riemannian case, we define the k-mean curvatures H_k of the immersion via the formula

$$\binom{n}{k} H_k = (-1)^k S_k.$$

and we introduce the Newton operators $P_k : T\Sigma \to T\Sigma$, inductively defined by

$$P_0 = I, \quad P_k = \binom{n}{k} H_k I + AP_{k-1},\ k = 1, \ldots, n.$$

Using the Newton operators we can define the second order linear differential operators $L_k : C^\infty(\Sigma) \to C^\infty(\Sigma)$ associated to P_k by $L_k f = \mathrm{Tr}(P_k \circ \mathrm{hess} f)$. Notice that, analogously to the Riemannian case, spacelike hypersurfaces of constant higher order mean curvature in Lorentzian spaceforms are critical points of some area functionals for volume preserving variations (see [BO] for more details).
(i) In [AIR2] we focus on uniqueness results for compact and for complete spacelike hypersurfaces of *generalized Robertson-Walker* (GRW) spacetimes having constant higher order mean curvature. Many works have appeared where uniqueness results are studied in the case where the ambient space is a GRW, which is nothing but a Lorentzian warped product space $-I \times_\rho \mathbb{P}^n$ with one dimensional basis. For instance, in [ARS1] Alías, Romero and Sánchez studied the problem of uniqueness of spacelike hypersurfaces in GRW spacetimes in the CMC case. In particular, it is proved that, when the spacetime obeys the so-called *timelike convergence condition*, then every compact CMC spacelike hypersurface must be totally umbilical and, in most of the cases, it must be a spacelike slice. In [ARS2], the same authors also observed that these results can be obtained replacing the timelike convergence condition by a weaker one, the so-called *null convergence condition*. Later on, the same problem was considered by Montiel [Mo2] that classified totally umbilical spacelike CMC hypersurfaces and proved that the only compact CMC spacelike hypersurfaces in a GRW spacetime obeying the null convergence condition are the spacelike slices, unless in the case where the spacetime is a de Sitter space and the hypersurface is a round umbilical hypersphere. Moreover, he also proved a uniqueness result for hypersurfaces of constant scalar curvature. In this circle of ideas, in [AC] Alías and Colares extended these results to compact spacelike hypersurfaces of constant k-mean curvature, $2 \le k \le n$, in proper GRW spacetimes (that is spacetime that cannot be written as trivial products, not even locally) obeying either the null convergence condition or under a milder assumption on the warping function ρ. For what concern the complete non-compact case, uniqueness results have been proved by Alías and Montiel in [AM] for complete CMC spacelike hypersurfaces in GRW spacetimes and, later on, by Romero and Rubio, in [RR], and by Caballero, Romero and Rubio, in [CRR], for CMC surfaces in GRW spacetimes. As a contribution to the theory we study the uniqueness problem for complete non-compact hypersurfaces of constant higher order mean curvature, generalizing the results in [AC] to the non-compact case. For such generalizations, it is employed the version of the Omori-Yau maximum principle for trace-type differential operator that was developed in [AIR], and the concept of parabolicity for divergence-type operators.

(ii) In [I], motivated by recent results on the topic of comparison theory and geometric analysis of the Lorentzian distance function proven in [AHP], we derive new Hessian and Laplacian comparison theorems for the Lorentzian distance function in a spacetime with sectional (or Ricci) curvature bounded by a suitable radial function. As an application of those comparison results, we study the Lorentzian distance function restricted to a spacelike immersed hypersurface bounded in a suitable spacetime, in the sense that the Lorentzian distance from a fixed point to the hypersurface is bounded from above. Using the comparison theorems previously proved jointly with the Omori-Yau maximum principle, we obtain sharp estimates for the mean curvature of such hypersurfaces provided that either the Ricci tensor of the ambient spacetime is bounded from below on timelike directions or the sectional curvatures of all timelike planes of the spacetime are bounded from below by the radial function, generalizing the mean curvature estimates obtained in [AHP]. Moreover, with the aid of the generalized Omori-Yau maximum principle for elliptic trace-type operators we are also able to obtain estimates for the higher order mean curvatures of the hypersurface generalizing the ones obtained in [AA1] and [AA2]. Finally, a Bernstein-type result for the Lorentzian distance function is given in case the spacetime has constant sectional curvature.

(iii) In [GI] we deal with height estimates for spacelike hypersurfaces Σ^n immersed with constant k-mean curvature, $1 \leq k \leq n$, in a GRW spacetime $-I \times_\rho \mathbb{P}^n$ and with boundary contained in a slice $\{s\} \times \mathbb{P}^n$. Recently the study of a priori estimates for the height of constant mean curvature compact spacelike graphs or, more generally, compact spacelike hypersurfaces with boundary, has become the subject of a rapidly increasing research. This is motivated by the fact that these estimates turn out to be a very useful tool in order to investigate existence and uniqueness results for complete spacelike hypersurfaces with constant mean curvature, as well as to obtain information on the topology at infinity of such hypersurfaces.

A priori estimates for the height of constant mean curvature compact spacelike hypersurfaces in the Lorentz-Minkowski spacetime $\mathbb{L}^{n+1}$ and with boundary on a spacelike hyperplane, were first obtained by López in [Lo] in case $n = 2$ and were later extended by Lima, [L], to any n . Later on, in [Mo3], Montiel obtained height estimates for compact spacelike graphs in the steady state space, which is an open region of the de Sitter spacetime, and he applied them to prove some existence and uniqueness theorems for complete spacelike hypersurfaces in the de Sitter space with constant mean curvature $H > 1$ and prescribed asymptotic future boundary. Later on, Colares and Lima, [CL], were able to generalize these estimates to the case of compact spacelike hypersurfaces of positive constant k-mean curvature in Lorentzian product spaces $-\mathbb{R} \times \mathbb{P}^n$ satisfying some energy condition and with boundary contained in a slice. These estimates have the important feature that they only depend on the k-mean curvature of the hypersurface and on a bound on the hyperbolic angle between the future-pointing unit normal vector field and the coordinate vector field induced by the universal time on $-\mathbb{R} \times \mathbb{P}^n$. Due to this feature, Colares and Lima were able to apply them to the study of topological properties of complete spacelike hypersurfaces of positive constant mean curvature. In [GI] we aim at completing the picture described above by considering compact spacelike hypersurfaces with boundary immersed in GRW spacetimes. Controlling the mean curvature of the hypersurface in terms of the mean curvature of the slices and imposing suitable conditions on the geometry of the ambient spacetime (that is, on the warping function and on the curvature of the fiber), we are able to extend the results in [CL] to compact spacelike hypersurfaces with constant k-mean curvature ($1 \leq k \leq n$) in a GRW spacetime and with boundary contained in a slice. As an application, we obtain some information on the topology at infinity of complete spacelike hypersurfaces of constant k-mean curvature properly immersed in a spatially closed generalized Robertson-Walker spacetime. Finally, using a version of the Omori-Yau maximum principle for the Laplacian and for more general elliptic trace-type differential operators, some non-existence results are also obtained.

- **Totally biharmonic submanifolds**([IM]).
The notion of harmonicity of manifold-valued maps was first introduced in the mid 1960's by J. Eells and J.H. Sampson, [ES]. Indeed, let $C^\infty(M, N)$ be the space of smooth maps $\varphi : (M, g) \to (N, h)$ between two Riemannian manifolds. A map φ in $C^\infty(M, N)$ is called

harmonic if it is a critical point of the energy functional

$$E : C^\infty(M, N) \to \mathbb{R}, \quad E(\varphi) = \frac{1}{2}\int_\Omega |\mathrm{d}\varphi|^2 v_g$$

and is characterized by the vanishing of the tension field $\tau(\varphi) = \mathrm{Tr}\nabla\mathrm{d}(\varphi)$. A natural generalization of harmonic maps can be given by considering the functional obtained integrating the square of the norm of the tension field. More precisely, let $\varphi \in C^\infty(M, N)$. One can define the *bienergy* of φ as

$$E_2(\varphi) = \frac{1}{2}\int_\Omega |\tau(\varphi)|^2 v_g.$$

A map φ is then said to be *biharmonic* if it is a critical point of the bi-energy functional $E_2 : C^\infty(M, N) \to \mathbb{R}$. In the last decade there has been a growing interest in the theory of biharmonic maps (see `people.unica.it/biharmonic` for a list of publications on the topic) which can be divided in two main research directions. The differential geometric aspect has mainly dealt with the construction of examples and classification results. On the other hand, there is also an analytic aspect from the point of view of PDE. Indeed biharmonic maps are solutions of a fourth order strongly elliptic semilinear PDE.

In [IM] we concentrated in the differential geometric aspect. In particular, we introduced the notion of totally biharmonic submanifolds: An isometrically immersed submanifold (M, g) in the Riemannian manifold (N, h) is called a *totally biharmonic submanifold* if all the geodesics of M are biharmonic curves of N. When the codimension is 1, we derive necessary and sufficient conditions, expressed explicitly in terms of the second fundamental form of M^n in N^{n+1}, for a geodesic of M^n to be biharmonic in N^{n+1}. Then we obtain the classification of totally biharmonic surfaces in the space form $N^3(c)$ by proving that if $c \le 0$, then M^2 is a totally geodesic surface; if $c > 0$ and letting $N^3(1) = \mathbb{S}^3(1)$, then M^2 is either a totally geodesic surface or one of the following two surfaces: part of the Clifford torus $\mathbb{S}^1(1/\sqrt{2}) \times \mathbb{S}^1(1/\sqrt{2})$, or part of the sphere $\mathbb{S}^2(1/\sqrt{2})$. Additionally, using the orthonormal basis of left-invariant vector fields and the left-invariant metric on the three-dimensional Heisenberg group H^3, via direct computation, we show that on an immersed right cylinder in H^3, there is a family of geodesics of the cylinder which are biharmonic curves of H^3.

Bibliography for the research activity

AEG J.A. Aledo, J.M. Espinar, and J.A. Gálvez, *Height estimates for surfaces with positive constant mean curvature in* $\mathbb{M}^2 \times \mathbb{R}$, Illinois J. Math. **52** (2008), no. 1, 203–211.

A1 A. D. Alexandrov, *Uniqueness theorems for surfaces in the large. VII*, Vestnik Leningrad. Univ., **15** (1960), no. 7, 5–13.

A2 A. D. Alexandrov, *A characteristic property of spheres*, Ann. Mat. Pura Appl., **58** (1962), no. 4, 303–315.

AA1 J. A. Aledo, L. J. Alías, *On the curvatures of bounded complete spacelike hypersurfaces in the Lorentz-Minkowski space*, Manuscripta Math., **80** (2000), no. 1-3, 51–58.

AA2 J. A. Aledo, L. J. Alías, *On the curvatures of complete spacelike hypersurfaces in de Sitter space*, Geom. Dedicata, **101** (2000), no. 3, 401–413.

AC L. J. Alías, A. G. Colares, *Uniqueness of spacelike hypersurfaces with constant higher order mean curvature in generalized Robertson-Walker spacetimes*, Math. Proc. Cambridge Philos. Soc. **143** (2007), no. 3, 703–729.

AD1 L. J. Alías, M. Dajczer, *Uniqueness of constant mean curvature surfaces properly immersed in a slab*, Comment. Math. Helv. **81** (2006), no. 3, 653–663.

AD2 L. J. Alías, M. Dajczer, *Constant mean curvature hypersurfaces in warped product spaces*, Proc. Edinb. Math. Soc. (2) **50** (2007), 511–526.

AHP L. J. Alías, A. Hurtado, V. Pamer, *Geometric analysis of Lorentzian distance function on spacelike hypersurfaces*, Trans. Amer. Math. Soc. **326** (2010), no. 10, 5083–5106.

AM L. J. Alías, S. Montiel, *Uniqueness of spacelike hypersurfaces with constant mean curvature in generalized Robertson-Walker spacetimes*, World Sci. Publ., River Edge, NJ, 2002.

ARS1 L. J. Alías, A. Romero, M. Sánchez, *Uniqueness of complete spacelike hypersurfaces of constant mean curvature in generalized Robertson-Walker spacetimes*, Gen. Relativity Gravitation **27** (1995), no. 1, 71–84.

ARS2 L. J. Alías, A. Romero, M. Sánchez, *Spacelike hypersurfaces of constant mean curvature and Calabi-Bernstein type problems*, Tohoku Math. J. (2) **49** (1997), no. 3, 337–345.

BE D. Bakry, and M. Émery, *Diffusions hypercontractives*, Séminaire de probabilités, XIX, 1983/84, Lecture Notes in Math. **1123**, 177–206.

BC J. L. M. Barbosa, A. G. Colares, *Stability of hypersurfaces with constant r-mean curvature*, Ann. Global Anal. Geom. **15** (1997), no. 3, 277–297.

BO J. L. M. Barbosa, V. Oliker, *Spacelike hypersurfaces with constant mean curvature in Lorentz space*, Mat. Contemp. **4** (1993), 27–44, VIII School on Differential Geometry (Portuguese) (Campinas, 1992).

BMR B. Bianchini, L. Mari, M. Rigoli, *Spectral radius, index estimates for Schrödinger operators and geometric applications*, J. Funct. Anal., **256** (2009), no. 6, 1769–1820.

CRR M. Caballero, A. Romero, R. M. Rubio, *Constant mean curvature spacelike surfaces in three-dimensional generalized Robertson-Walker spacetimes*, Lett. Math. Phys. **93** (2010), no. 1, 85–105.

dCZ M. P. do Carmo, D. T. Zhou, *Eigenvalue estimate on complete noncompact Riemannian manifolds and applications.*, Trans. Amer. Math. Soc. **351** (1999), no. 4, 1391–1401.

CMD X. Cheng, T. Mejia, and D. Zhou *Stability and compactness for complete f-minimal surfaces.* arXiv:1210.8076 (2012)

CR X. Cheng and H. Rosenberg, *Embedded positive constant r-mean curvature hypersurfaces in $M^m \times$* **R**, An. Acad. Brasil. Ciênc. **77** (2005), no. 2, 183–199.

CL A.G. Colares and H.F. de Lima, *Space-like hypersurfaces with positive constant r-mean curvature in Lorentzian product spaces*, Gen. Relativity Gravitation **40** (2008), no. 10, 2131–2147.

CoMi T. Colding, W. Minicozzi, *Generic mean curvature flow I; generic singuarities.* Ann. of Math. **175** (2012), 755-833.

ES J. Eells, J. H. Sampson, *Harmonic mappings of Riemannian manifolds*, Amer. J. Math **86** 1964 109–160.

E M. F. Elbert, *Constant positive 2-mean curvature hypersurfaces*, Illinois J. Math. **46** (2002), no. 1, 247–267.

Esp J. M. Espinar *Manifolds with density, applications and gradient Schrödinger operators.* arXiv:1209.6162v6. (2012)

F E. M. Fan, *Topology of three-manifolds with positive P-scalar curvature.* Proc. Amer. Math. Soc. **136** (2008), no. 9, 3255–3261.

Gr1 A. Grigor'yan, *On the existence of positive fundamental solutions of the Laplace equation on Riemannian manifolds.* Mat. Sb. (N.S.) **128** (1985), no. 3, 354–363. http://m.iopscience.iop.org/0025-5734/56/2/A05/pdf/0025-5734_56_2_A05.pdf.

Gr2 A. Grigor'yan, *Analytic and geometric background of recurrence and non-explosion of the Brownian motion on Riemannian manifolds.* Bull. Amer. Math. Soc. (N.S.) 36 (1999), no. 2, 135–249.

G M. Gromov, *Isoperimetry of waists and concentration of maps.* Geom. Funct. Anal. **13** (2003), no. 1, 178-215.

HLR D. Hoffman, J.H.S. de Lira, and H. Rosenberg, *Constant mean curvature surfaces in $M^2 \times \mathbf{R}$*, Trans. Amer. Math. Soc. **358** (2006), no.2, 491–507 (electronic).

Ju C. Hussey, *Classification and analysis of low index Mean Curvature Flow self-shrinkers*, Thesis (Ph.D.)–The Johns Hopkins University (2012).

KKMS N.J. Korevaar, R. Kusner, W. H. Meeks, III, and B. Solomon, *Constant mean curvature surfaces in hyperbolic space*, Amer. J. Math. **114** (1992), no. 1, 1–43.

L H.F. de Lima, textslA sharp height estimate for compact spacelike hypersurfaces with constant r-mean curvature in the Lorentz-Minkowski space and application, Differential Geom. Appl. **26** (2008), no. 4, 445–455.

Lo R. López, *Area monotonicity for spacelike surfaces with constant mean curvature*, J. Geom. Phys. **52** (2004), no. 3, 353–363.

Mo1 S. Montiel, *Unicity of constant mean curvature hypersurfaces in some Riemannian manifolds*, Indiana Univ. Math. J. **48** (1999), no. 2, 711–748.

Mo2 S. Montiel, *Uniqueness of spacelike hypersurfaces of constant mean curvature in foliated spacetimes*, Math. Ann. **314** (1999), no. 3, 529–553.

Mo3 S. Montiel, *Complete non-compact spacelike hypersurfaces of constant mean curvature in de Sitter spaces*, J. Math. Soc. Japan **55** (2003), no. 4, 915–938.

Mo-Notices F. Morgan, *Manifolds with density*, Notices Amer. Math. Soc. **52**, (2005), no. 8, 853–858.

PRS S. Pigola, M. Rigoli, A. G. Setti, *Maximum principles on Riemannian manifolds and applications*, Mem. Amer. Math. Soc. 174 (2005), no. 822, x+99.

Re R. C. Reilly, *Variational properties of functions of the mean curvatures for hypersurfaces in space forms*, J. Differential Geometry **8** (1973), 465–477.

Ro H. Rosenberg, *Hypersurfaces of constant curvature in space forms*, Bull. Sci. Math. **117** (1993), no. 2, 211–239.

RR A. Romero, R. M. Rubio, *On the mean curvature of spacelike surfaces in certain three-dimensional Robertson-Walker spacetimes and Calabi-Bernstein's type problems*, Ann. Global Anal. Geom. **37** (2010), no. 1, 21–31.

Sc L. Shahriyari, *Translating graphs by mean curvature flow*, ProQuest LLC, Ann Arbor, MI, 2013, Thesis (Ph.D.)–The Johns Hopkins University.

Sm K. Smoczyk, *A relation between mean curvature flow solitons and minimal submanifolds,* Math. Nachr. **229** (2001), 175–186.

WW G. Wei and W. Wylie, *Comparison geometry for the Bakry-Emery Ricci tensor*, J. Differential Geom., **83**, (2009) no. 2, 377–405.

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